國立中央大學97學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 乙組　　科目：微積分　　共 1 頁　第 1 頁

*請在試卷答案卷（卡）內作答

參考用

1. (10%) Please examine the following series for convergence or divergence with proof:

$$\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\frac{9}{25}+\cdots=\sum_{k=1}^{\infty}\frac{2k+1}{(k+1)^2}$$

2. (20%) Please evaluate the following integrals.

   (a) (10%) $\int \frac{\sqrt{2+3\sqrt{x}}}{\sqrt{x}}dx$

   (b) (10%) $\int_1^2 \frac{1}{x^2}\sqrt{1-\frac{1}{x}}dx$

3. (20%) By considering $\frac{d^n}{dy^n}\int_0^1 x^y dx$, please find $\int_0^1 (\log x)^n\, dx$.

4. (15%) Suppose that $g: R^2 \to R$. For a double integral $\int_0^t\int_0^u g(s,u)\,ds\,du$, after changing the order of this double integral, what would be the new invervals for the double integral? You just need to find what the three question marks are in the second double integral for this question.

$$\int_0^t\int_0^u g(s,u)\,dsdu=\int_0^?\int_?^? g(s,u)\,duds$$

5. (15%) A call option gives its holder the right to buy an asset $S_\bullet$ at time T with price K. Denote the price of the asset at time t as S_t. If $S_T > K$ at option maturity date T, then $C_T = S_T - K$; if $S_T \leq K$ at option maturity date T, then $C_T = 0$, where C_T is the option payoff at maturity time T. Assuming that the probability density function of the asset price is $f(S_\bullet)$, then the call option price at current time 0 is equal to

$$C_0 = e^{-rT}\int_K^{\infty}(S_T-K)\,f(S_T)\,dS_T$$

   where r, K and T are constant.

   Please calculate the second derivatives of the call price with respect to K, i.e. $\frac{\partial^2 C_0}{\partial K^2}$.

6. (20%) The put option price at current time $t=0$ is $P_0 = Ke^{-rT}N(-d_2) - S_0N(-d_1)$, where $d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$, r, σ, K and T are constant.

   $N(x)$ is the cumulative distribution function of the standard normal distribution, whose probability density function is $n(x)$. By definition, $\frac{\partial N(x)}{\partial x} = n(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$.

   Please find the first derivatives of the put price with respect to S_0, i.e. $\frac{\partial P_0}{\partial S_0}$.