1. (20%) Solve the following recursion relation.

(a) $s_n = s_{n-1} + 2s_{n-2}$ for $n \in N$ and $s_0 = s_1 = 3$

(b) $s_n = s_{n-1} + s_{n-2}$ for $n \geq 2$ and $s_0 = s_1 = 1$.

where N is the set of all nonnegative integers.

2. (20%) A deck of cards consists of four suits called clubs, diamonds, hearts and spades. Each suit consists of thirteen cards with values A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. There are four cards of each value, one from each suit. A poker hand is a set of 5 cards from a 52-card deck of cards. The order in which the cards are chosen is irrelevant. A straight consists of five cards whose values form a consecutive sequence such as 8,9,10,J,Q. The ace A can be at the bottom of a sequence A,2,3,4,5 or at the top of a sequence 10,J,Q,K,A Poker hands are classified into disjoint sets as follows.

(a) Royal flush:10,J,Q,K,A all in the same suit.

(b) Straight flush:A straight all in the same suit that is not a royal flush.

(c) Four of a kind:Four cards in the hand have the same value.

(d) Full house:Three cards of one value and two cards of another value.

(e) Flush:Five cards all in the same suit, but not a royal or straight flush.

(f) Straight: A straight that is not a royal or straight flush.

(g) Three of a kind:Three cards of one value, a fourth card of a second value and a fifth card of a third value.

(h) Two pairs:Two cards of one value, two more cards of a second value and the remaining card a third value.

(i) One pair:two cards of one value, but not classified above.

(j) None: none of the above.

Please count the number of poker hands for each set from (a) to (j)

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3. (20%) Let N be the set of all nonnegative integers. On the set $N \times N$ define $\langle m,n \rangle \sim \langle k,l \rangle$ if $m+l=n+k$.

(a) Show that ~ is an equivalent relation on $N \times N$.

(b) Draw a sketch of $N \times N$ that shows the equivalent classes.

4. (20%) Prove that

if $x_1, x_2, \ldots, x_n$ are all positive numbers, then

$(x_1 x_2 x_3 \ldots x_n)^{\frac{1}{n}} \leq \frac{x_1+x_2+\ldots+x_n}{n}$.

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5.(20%) Fill in the following blanks

(a) The complete bipartite graph $K_{m,n}(m+n \geq 3)$ has a hamiltonian path if and only if _____ _______.

(b) A graph is a tree if and only if there is _______. __ between every pair of vertices in graph.

(c) Let G be a connected graph with n vertices and m edges ($m>n$). The maximum number of edges that can be broken with connection among all vertices still possible is __________.

(d) Let $S_n=\{1,2,...,n\}$. Define $x+y=\max\{x,y\}$ and $x \bullet y=\min\{x,y\}$. It is possible to define 0,1, and ' so that $(S_n,+,\bullet,',0,1)$ is a Boolean algebra if and only if $n=$____. ____.

(e) An optimal expression equivalent to $(x \vee y)' \vee z \vee x(yz \vee y'z')$ is ________.

6.(10%) Let G be an undirected graph with k components, n vertices and m edges. Prove that $m \geq n-k$.

7.(10%) Simplify the expression $[\ln n + O(\frac{1}{n})][n + O\sqrt{n}\,]$.

8.(10%) Let A be a set of cities and R be a binary relation on A so that the ordered pair (a,b) is in R if there is a communication link from city a to city b for the transmission of messages. Let R_1 be the transitive extension of R, R_2 be the transitive extension of R_1 and R^* be the transitive closure of R. Explain the practical meanings of R_1, R_2 and R^*.