所別:資訊工程學系碩士班 科目:離散數學與線性代數 共 2 頁 軟體工程研究所碩士班

※ 請務必按照題號次序寫在答案紙上,否則將嚴重失分。

- 1.(20%) Consider the following algorithm, which shuffles a list of integers. The algorithm assumes that the list size, n, is a power of 3.
 - 1: **shuffle**($\{a_0, a_1, ..., a_n\}$) 2: if n == 1 return $\{a_0\}$

 - 3: m = n/3
 - 4: $b = \text{shuffle}(\{a_0, ..., a_{m-1}\})$ # the result is { b_0, \dots, b_{m-1} }
 - 5: $c = \text{shuffle}(\{a_m, ..., a_{2m-1}\})$ # the result is $\{c_0, \ldots, c_{m-1}\}$
 - 6: $d = \text{shuffle}(\{a_{2m}, \dots, a_{3m-1}\})$ # the result is { d_0, \ldots, d_{m-1} }
 - 7:
 - 8: $e = \{c_0, \ldots, c_{m-1}, d_0, \ldots, d_{m-1}, b_0, \ldots, b_{m-1}\}$

10: return e

11 end shuffle

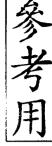
Answer the following questions according to the different assumptions:

- (a)(10%) If line 8 use n copies to form the new list, what is the recurrence relation that counts the number of data copies for this algorithm? And estimate the complexity using big- θ notation.
- (b)(10%) If instead the list is implemented by pointers and line 8 therefore only needs 3 pointers movements. What is the recurrence relation that counts the number of pointer movements for this algorithm? And estimate the complexity using big- θ notation.
- 2.(5%) What is the generating function for $\{a_k\}$, where a_k represents the number of ways to make change for kdollars using 1 dollor, 5 dollor, 10 dollor, and 20 dollors bills?
- (3.(15%)) Let $S = \{2, 3, 5, 7, 11, 13, 17, 19\}$ be the set of prime numbers less than 20. If A is a subset of S, we can form the sum and product of the elements of A. For example, if $A = \{7, 11, 13\}$, then the associated sum is 7 + 11 + 13 = 31 and the associated product is 7(11)(13) = 1001.
 - (a)(8%) Use the Pigeon-Hole Principle to show that there are four nonempty subsets of S with the same sum.
 - (b)(7%) Are there two nonempty subsets of S with the same product? Explain.
- 4.(10%) For integers a and b, define $a \sim b$ if 3a + 4b = 7n for some integer n.
 - (a)(5%) Prove that \sim defines an equivalence relation.
 - (b)(5%) Find the equivalence class of 0.

(還有第二頁)

:背面有試題

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※ 請務必按照題號次序寫在答案紙上,否則將嚴重失分。

- 5.(10%) Find a subset of the following four vectors $v_1 = (1,0,1,1)$, $v_2 = (-3,3,7,1)$, $v_3 = (-1,3,9,3)$, and $v_4 = (-5,3,5,-1)$, that forms a basis for the space spanned by these vectors.
- 6.(10%) Find a basis for the nullspace of matrix A, where $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & 1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$.
- 7.(5%) Determine whether the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$, $v_3 = (3, 2, 1)$ form a linearly dependent set or a linearly independent set.
- 8.(18%) True and false (每小題答對給 3 分,答錯扣 3 分,不答 0 分;本題總分≥0)
 - (a) If square matrix A has one zero column, then $A^{T}A$ is not diagonalizable.
 - (b) Every orthonormal set in \mathbb{R}^n is linearly independent.
 - (c) If A is a square matrix with n orthonormal columns, then the rows of A must be an orthonormal basis for R^n .
 - (d) For an inconsistent linear system A x = b, we can find its least-squares solution $(A^T A)^{-1} A^T b$. However, if $(A^T A)$ is not invertible, there is no least-squares solution.
 - (e) The quadratic form $2x_1^2 + 10x_1x_2 + 2x_2^2$ can be transformed into $7y_1^2 3y_2^2$ with no cross-product term.
 - (f) If A is a square matrix, then det A is the product of the singular values of A.
- 9.(7%) For a linear transform $x \mapsto Ax$ with $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$. Find a unit vector x at which Ax has maximum length and compute the length.

(題目到此為止)