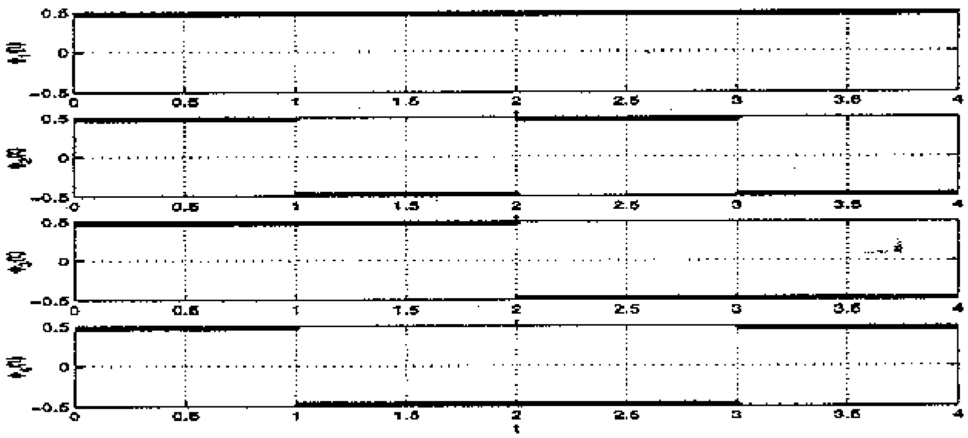
- 請從六個試題中選五題作答。若作答六題，以得分較低的五題計分。

1. Consider the set of four functions $\phi_1(t), \phi_2(t), \phi_3(t)$, and $\phi_4(t)$ shown below. The signal

$$x(t) = \begin{cases} 4t, & 0 \le t \le 4 \\ 0, & otherwise \end{cases}$$

   (a) Find the expression for $x_a(t)$ over the interval $0 \le t \le 4$ in terms of $\phi_1(t), \phi_2(t), \phi_3(t)$, and $\phi_4(t)$, (i.e., $x_a(t) = \alpha_1\phi_1(t) + \alpha_2\phi_2(t) + \alpha_3\phi_3(t) + \alpha_4\phi_4(t)$) such that the intergral squared error $\epsilon_N = \int_0^4 \|x(t) - x_a(t)\|^2 \, dt$ (ISE) is minimized. [15 pts.]

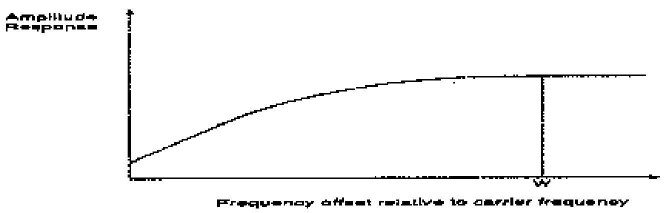
   (b) Calculate the minimium ISE. [5 pts.]



2. We have a FM signal: $x_c(t) = A_c \cos[2\pi f_c t + \phi(t)]$, where $\phi(t) = 30\pi \int_0^t 4\sin(40\pi\alpha)\, d\alpha$, and $f_c = 1000Hz$

   (a) Find the value of the modulation index [5 pts.]

   (b) What is the reason we use the *deemphasis* filter in the FM demodulation? [5 pts.]



   (c) If we have the response of a *deemphais* filter as the above(W is the bandwidth of the message), how do you design your *preemphasis* filter (Draw the amplitude response of the *preemphasis* filter approximately)? Explain your design. [5 pts.]

   (d) What is the threshold effect in the FM discriminator? Explain briefly why the frequency-compressive feedback loop can be used for the threshold extension. [5 pts.]

3. The output of an operational amplifier is

$$Y = a_1X_1 + a_2X_2 + a_3X_3,$$

where $X_i, i = 1, 2, 3$, are inputs and a_i are some constants.

(Case 1) If X_i are independently Gaussian distributed with means $E[X_i] = m_i$ and variances $Var[X_i] = \sigma_i^2$.

(Case 2) If X_i are independently Poisson distributed with the probability mass functions

$$\Pr(X_i = k) = \frac{\Lambda_i^k e^{-\Lambda_i}}{k!}, \qquad k = 0, 1, 2, 3, \ldots$$

Find the distribution of Y for both cases. [20 pts.]

注意：背面有試題

4. The random variable Y is Gaussian distributed with mean 0 and variance 1. Y is processed through a shaper.

   (Case 1) The output of the shaper is $Z = \tanh Y$. (Note: tanh denotes hyperbolic tangent function.)

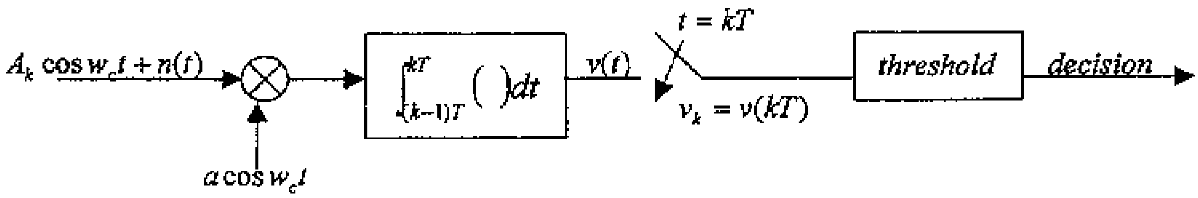
   (Case 2) The output of the shaper is $Z = Y^2$.

   Find the distribution of Z for both cases. [20 pts.]

5. Consider the coherent receiver shown below. The received signal is $A_k cos\omega_c t + n(t)$, where $n(t)$ is a zero-mean white Gaussian noise with double-sided power spectral density $\frac{N_0}{2}$. The amplitude A_k carries the information bit at time k. Let v_k denote the output of the sampler at time k, and T denote the signalling interval. Assume that $\omega_c = \frac{2\pi m}{T}$, m is an integer.

   (a) If $A_k \in \{A, -A\}$, and the value of a is chosen such that v_k is equal to $1+N$ or $-1+N$, where N is a zero-mean random variable. Let $\sigma^2 = E\{N^2\}$, (where $E\{\}$ is the expectation operator). Represent $\frac{E_s}{N_0}$ in terms of σ, where E_s denote the energy per symbol. [10 pts.]

   (b) The function of the "threshold" box is to decide the value of A_k. If $A_k \in \{A, -\frac{A}{2}\}$ and $a = 2$, what is the error probability with the optimum threshold? (Use Q or erfc functions.) [10 pts.]



6. Consider the noncoherent receiver shown below. Let x_k and y_k denote the outputs of the samplers at time k. Let T denote the signalling interval. Assume that $\omega_c = \frac{2\pi m}{T}$, m is an integer.

   (a) Represent $E\{x_k\}$ and $E\{y_k\}$ in terms of A, ϕ_k, θ and T (where $E\{\}$ is the expectation operator). [10 pts.]

   (b) Assume that θ is a constant and unknown at the receiver. Let ϕ_k denote the modulation phase at time k and $\phi_k = (\phi_{k-1} + \Delta\phi_k)$ *modulo* 2π, where $\Delta\phi_k$ is the data phase at time k. Assume that $\Delta\phi_k \in \{-\frac{\pi}{2}, 0, \frac{\pi}{2}\}$. In the "decision logic" box, the test statistic is used to detect the value of $\Delta\phi_k$. There are two possible test statistics: $l_1 = x_k x_{k-1} + y_k y_{k-1}$ and $l_2 = x_k y_{k-1} - y_k x_{k-1}$. Which test statistic is suitable? Explain the reason. [10 pts.]