

注意：考試開始鈴響前，不可以翻閱試題

台灣聯合大學系統 107 學年度學士班轉學考試題

考試科目：微積分

組別：A3、A4、A7

參考用

—作答注意事項—

1. 作答中如發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
2. 請核對答案卷（卡）上之准考證號、考試科目是否正確。
3. 本考科禁止使用計算器。
4. 請在答案卷（作答區內）作答。
5. 考生限在作答區內作答，不可書寫姓名、准考證號或與作答無關之其他文字或符號。
6. 答案卷用盡不得要求增加。
7. 答案卷限用藍筆或黑色鋼筆、原子筆或鉛筆書寫；答案卡限用 2B 軟心鉛筆畫記，如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果考生自行負責。
8. 因字跡潦草或作答未標明題號等情事，致評閱人員無法辨識答案者，該部分不予計分。

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲）部分，共 8 題，命題型態為填充題，必須以填充題形式將答案寫在答案卷第一頁，倘若答案被包含在演算過程中，將被視為試算草稿，無法採計分。

參考用

1. Find the critical number of $y = 1 - \frac{4}{\pi^2}(\tan^{-1} x)^2$. Answer : _____

2. Determine the limits of integration where $a \leq b$ such that $\int_a^b (x^2 - 16) dx$ has minimal value. Answer : _____

3. Evaluate $\int_{-\infty}^{\infty} \frac{e^x}{1 + e^{2x}} dx$. Answer : _____

4. Find the slope of the surface $f(x, y) = (x^3 + y^3)^{1/3}$ at the point $(0, 0)$ in the y -direction. Answer : _____

5. Find the surface area of the portion of the plane $z = 4 - 2x - 2y$ that lies above the circle $x^2 + y^2 \leq 1$ in the first quadrant. Answer : _____

6. Find an equation of the tangent plane to the paraboloid $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (u^2 + v^2)\mathbf{k}$ at the point $(1, 2, 5)$. Answer : _____

7. Evaluate the integral $\int_0^{\infty} \int_0^{\infty} \frac{1}{(1 + x^2 + y^2)^2} dx dy$. Answer : _____

8. Use a change of variables to find the volume of the solid region lying below the surface $z = \sqrt{(x + 4y)(x - y)}$ and above the plane region R : region bounded by the parallelogram with vertices $(0, 0)$, $(1, 1)$, $(5, 0)$ and $(4, -1)$: Answer : _____

乙、計算、證明題：共 3 大題，每大題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. A heat-seeking particle is located at the point $(-1, 2)$ on a metal plate whose temperature at (x, y) is $T(x, y) = 64 - 2x^2 - y^2$. (a) (6 分) In what direction from $(-1, 2)$ does the temperature increase most rapidly? What is this rate of increase? (b) (6 分) Find the path of the particle as it continuously moves in the direction of maximum temperature increase.

2. Determine if the given series converges or diverges. Explain your reasoning.

a. (6 分) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{n+3}\right)^n$ b. (6 分) $\sum_{n=1}^{\infty} \frac{e^{2/n}}{n^2}$

3. Find the maximum value of $\int_C y^3 dx + (27x - x^3) dy$, where C is any circle in the xy -plane, oriented counterclockwise.