1. (15%) For a linear system given by $\begin{cases} 3\cdot x_1 - 7\cdot x_2 - 2\cdot x_3 = -7 \\ -3\cdot x_1 + 5\cdot x_2 + 1\cdot x_3 = 5, \\ 6\cdot x_1 - 4\cdot x_2 + 0\cdot x_3 = 2 \end{cases}$

   (a) express this system in matrix form, i.e., $\mathbf{A}\cdot\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \vec{\boldsymbol{v}}$;

   (b) find the corresponding LU factorization, i.e., $\mathbf{A} = \underbrace{\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ ? & \mathbf{1} & \mathbf{0} \\ ? & ? & \mathbf{1} \end{bmatrix}}_{\mathbf{L}} \cdot \underbrace{\begin{bmatrix} ? & ? & ? \\ \mathbf{0} & ? & ? \\ \mathbf{0} & \mathbf{0} & ? \end{bmatrix}}_{\mathbf{U}}$;

   (c) find the solution of $\vec{\boldsymbol{x}}$.

2. (15%) For $\mathbf{A} = [\vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3] = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & -2 \\ 5 & 5 & 0 \end{bmatrix}$ with a known eigenvector $\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$,

   (a) find the eigenvalue λ_1 corresponding to eigenvector $\vec{v}_1$;

   (b) find the eigenvector $\vec{v}_2$ corresponding to eigenvalue $\lambda_2 = 5$;

   (c) the value (x, y) such that $\mathbf{A}\cdot\begin{bmatrix} x \\ 1 \\ y \end{bmatrix} = \vec{0}$.

3. (10%) For $\vec{y} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ and $\hat{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a_1\cdot\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_2\cdot\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, find $\min_{a_1,a_2}\|\vec{y}-\hat{y}\|$ and

   $\{a_1, a_2\}$, where $\|\vec{x}\| = \sqrt{\vec{x}^T\cdot\vec{x}}$ denotes the norm of the vector $\vec{x}$.

4. (10%) For an **inner product** definition given by $\langle f, g\rangle \equiv \int_{-1}^{1} f(t)\cdot g(t)dt$, and

   $s_1(t) = 1$, $s_2(t) = \begin{cases} 1, & 0 \le t \\ 0, & t < 0 \end{cases}$,

   (a) find the value of $\langle s_1, s_2\rangle$;

   注意:背面有試題

   (b) find $s(t) = s_2(t) + a_1\cdot s_1(t)$, $a_1 \in R$, i.e., the value of a_1 such that $\langle s_1, s\rangle = 0$.

國立中央大學111學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)

科目： 工程數學(線性代數、機率)

5. (10%) X is a binomial random variable with the probability mass function (PMF)

$$P_X(x) = \binom{4}{x}(1/2)^4.$$

Find the probability $P[X \le (\mu_X + \sigma_X)]$, where μ_X and σ_X denote the expected value and standard deviation of X, respectively.

6. (10%) X is a Gaussian random variable where the expected value and standard deviation are 0 and 4, respectively. Find the conditional expected value $E[X \mid A]$ given the event $A = \{X \ge 0\}$.

7. (15%) The 4-dimensional random vector $\mathbf{X} = [X_1\ X_2\ X_3\ X_4]'$ has probability density function (PDF)

$$f_{\mathbf{X}}(\mathbf{x}) = \begin{cases} 1 & 0 \le x_i \le 1, i = 1,2,3,4 \\ 0 & otherwise. \end{cases}$$

Find the expected value of X_1, $E[X_1]$.

8. (15%) Random variables X and Y have the joint probability density function (PDF)

$$f_{X,Y}(x,y) = \begin{cases} 2 & x \ge 0, y \ge 0, x + y \le 1, \\ 0 & otherwise. \end{cases}$$

Find the variance of X, $Var[X]$.

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