科目 微積分 類組別 A2 A3 A4 A5 A6

甲、計算、證明題:共3大題,每大題 12分,共36分。請在答案卷上詳細寫出計算及證明過程,否則不予計分。

- 1. Let $f(x,y) = \frac{x^3 + y^3}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Is it possible to define f(0,0) in a way that makes f continuous at the origin? Explain your answer and show your reasons. (Hint: Let $x = r \cos \theta$, $y = r \sin \theta$)
- 2. Evaluate the line integral by two methods: (a) directly and (b) using Green's Theorem.

$$\oint_C x\,dx + y\,dy,$$

where C consists of the line segments from (0,1) to (0,0) and from (0,0) to (1,0) and the parobola $y = 1 - x^2$ from (1,0) to (0,1).

3. Determine if the given series converges or diverges. Explain your answer and show your reasons.

(a)
$$\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n}).$$

(b)
$$\sum_{n=2}^{\infty} \frac{2}{n \ln n}$$

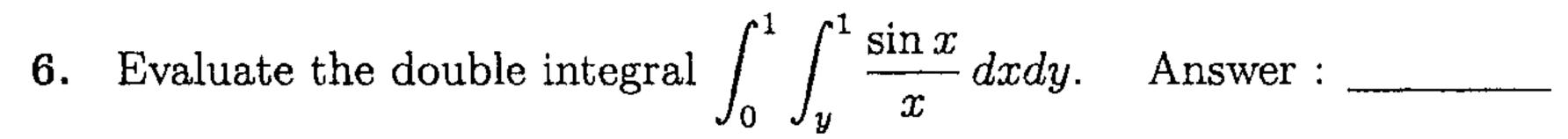
乙、填充題:共8題,每題8分,共64分。請僅就答案部分依題號順序寫在答案 卷上,不須呈現演算過程。

1. If
$$\lim_{x \to -2} \frac{f(x)}{x^2} = 1$$
, find $\lim_{x \to -2} \left(f(x) + \frac{f(x)}{x} \right)$. Answer:

2. Find
$$\lim_{x\to 0} \frac{1}{3x^2} \int_{x^2}^{0} \cos t \, dt$$
. Answer: ______

- 3. Find the rate of change of pressure at the point P = (1, 2, 0) in the direction of $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, where the pressure is given by $g(x, y, z) = xe^{-yz}$. Answer:
- 4. If $y = x^{\sin x}$, find the derivative of y with respect to x. Answer:

5. Evaluate
$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{0} \frac{4}{1+x^2+y^2} dxdy$$
. Answer:



7. Evaluate the definite integral
$$\int_{-1}^{1} \frac{e^{\tan^{-1}y}}{1+y^2} dy$$
. Answer: ______

